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Unsteady conjugate heat transfer for a single particle and in multi-particle systems at low Reynolds numbers

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Abstract—Two cases of the transient conjugate heat transfer in fluid–particle systems are analysed. The first refers to the single rigid spherical particle at particle Reynolds numbers greater than one. The subject of the second is the assemblages of rigid spherical particles at low particle Reynolds numbers, i.e. $Re \leqslant 20$. In this case the classical cell models are used to describe the hydrodynamics. In both cases the momentum and the heat balance equations are solved numerically. The velocity field is assumed to be steady, axisymmetric and laminar. The finite difference methods are used for discretization. The single particle problem is solved for $Re = 10$ (50), $Pe = 100.0$, and a wide domain of variation of the conductivity and volume heat capacity ratios. Simulation results reveal that the conductivity and heat capacity ratios affect the heat transfer remarkably. For the particle assemblages, the analysis is focused on the specific problems of the interphase heat transfer in packed beds at low Peclet numbers. The results show that the theoretical predictions of the unsteady conjugate heat transfer in cell models are not a good description of the packed bed experimental data. © 1997 Elsevier Science Ltd.

INTRODUCTION

A large number of studies have been published on the subject of momentum, heat and mass transport to a body of revolution. Clift *et al.* [1] give an extensive coverage of transport phenomena around a spherical particle. The majority of the theoretical studies concerning the heat transfer from a particle moving in another fluid, dealt with two limiting cases, for which the rate of transfer is controlled by the resistance, either of the dispersed phase (the internal problem) or of the continuous phase (the external problem). The conjugate heat and/or mass transfer from a spherical particle is the subject of relatively few theoretical and experimental studies. Due to the goal of this paper, the presentation of the previous contributions in solving the conjugate problem is restricted to the case of the rigid particle. Also, for the same reason, the free convection analyses are not mentioned here in detail. A recent analysis concerning this subject can be viewed in ref. [2].

The stagnant phases situation was solved numerically in ref. [3] and analytically in ref. [4]. In creeping flow the conjugate heat transfer was analysed in ref. [5] (the case of equal diffusivities in both phases), [6] (the ratio of the volume heat capacity is comprised between 0.0 and 2.0 and the particle temperature is considered uniform) and [7] (the conductivity and heat capacity ratios vary between 0.01 and 100.0). The moderate Reynolds numbers domain was solved by Nguyen *et al.* [8]. In ref. [8] the conductivity ratio varies between 1/3 and 3.0 and the volume heat capacity ratio takes only the values 1.0 and 1.0×10^{-3}.

The conjugate heat transfer in a multi-particle system was not analysed until now. The attention was focused only on the external problem [9, 10]. The hydrodynamical cell models [11, 12] predict, successfully, the pressure drop in packed beds in the voidage range 0.30–0.60 at low particle Reynolds number. At low values of the continuous phase Peclet numbers, the relations usually used to compute the interphase heat transfer in packed beds are not in good agreement with the experimental data (see for an illuminate discussion [13]). A possible explanation is proposed in ref. [14]. Other models of the packed bed, which take into consideration the temperature variation in both phases and an axial temperature gradient on the assemblage, are presented, for example, in refs [15] and [16].

This paper has a double aim. First, in the range of moderate *Re* numbers the effect of the conductivity ratio and the volume heat capacity ratio on the conjugate heat transfer from a single sphere is investigated. Second, using the cell models, the conjugate heat transfer in a packed bed is analysed and the values obtained for the asymptotic *Nu* number are compared with those usually used.

MATHEMATICAL MODEL

Consider a fluid particle system formed by a single particle or by an assembly of uniform rigid spheres of

NOMENCLATURE

a	radius of spherical particle
c_p	specific heat
Nu	instantaneous Nusselt number defined by equation (18)
Nu_θ	local Nusselt number defined by equation (17)
Pe	Peclet number, $2Ua/\alpha_2$
Pr	Prandtl number, ν_2/α_2
r	dimensionless radial distance in spherical coordinate system
Re	Reynolds number, $2Ua/\nu_2$
t	time
T	temperature
U	free stream velocity
V_R, V_θ	components of dimensionless velocity in spherical coordinate system
Z	dimensionless temperature $(T-T_\infty)/(T_0-T_\infty)$.

Greek symbols

α	thermal diffusivity
ε	voidage
Φ_h	the volume heat capacity ratio $(\rho_1 c_{p1})/(\rho_2 c_{p2})$
Φ_α	the thermal diffusivity ratio, α_1/α_2
Φ_λ	the thermal conductivity ratio, λ_1/λ_2
κ	defines the dimensionless radius of the outer hypothetical sphere
λ	thermal conductivity
ν	kinematic viscosity
θ	polar angle in spherical coordinate system
ρ	density
τ	dimensionless time, $\alpha t/a^2$.

Subscripts

ex	refers to the external problem
in	refers to the internal problem
1	sphere
2	fluid
∞	large distance from the spherical particle.

radius a. In the assembly the particles are considered fixed in space and equally spaced in the radial and longitudinal directions. Each sphere, with a surrounding spherical envelope of fluid, is uncoupled from the system and considered separately. The common assumptions, valid in both cases (i.e. single particle and cloud of particles):

(a) the flow field is steady, laminar and axisymmetric;
(b) the physical properties within the particle and in the surrounding medium, the particle shape and volume remain constant during the transfer,

are considered valid.

Under these assumptions, the unsteady conjugate heat transfer equations in dimensionless form, with the proper initial and boundary conditions, reads as follows:

the continuous phase

$$\frac{\partial Z_2}{\partial \tau_2} + \frac{Pe}{2}\left(V_R \frac{\partial Z_2}{\partial r} + \frac{V_\theta}{r}\frac{\partial Z_2}{\partial \theta}\right) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial Z_2}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial Z_2}{\partial \theta}\right) \quad (1)$$

the dispersed phase

$$\frac{\partial Z_1}{\partial \tau_1} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial Z_1}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial Z_1}{\partial \theta}\right) \quad (2)$$

the initial conditions

$$Z_1(r,\theta,0) = 1.0, \quad Z_2(r,\theta,0) = 0.0 \quad (3)$$

the boundary conditions

$$Z_1(0,\theta,\tau) < \infty \quad (4)$$

$$\frac{\partial Z_i}{\partial \theta}(r,0,\tau_i) = \frac{\partial Z_i}{\partial \theta}(r,\pi,\tau_i) = 0.0 \quad (5)$$

$$Z_2(\kappa,\theta,\tau_2) = 0.0 \quad (6)$$

$$\Phi_\lambda \frac{\partial Z_1}{\partial r}(1,\theta,\tau_1) = \frac{\partial Z_2}{\partial r}(1,\theta,\tau_2),$$

$$Z_1(1,\theta,\tau_1) = Z_2(1,\theta,\tau_2). \quad (7)$$

In the case of the single particle $\kappa = \infty$. For the assembly, κ depends on the packing voidage ε according to the relation

$$\kappa = (1-\varepsilon)^{-1/3}.$$

The dimensionless velocities are computed with the relations:

$$V_R = -\frac{1}{r^2 \sin\theta}\frac{\partial \psi}{\partial \theta}; \quad V_\theta = \frac{1}{r \sin\theta}\frac{\partial \psi}{\partial r}. \quad (8)$$

The dimensionless stream function, ψ, is the solution of the dimensionless Navier–Stokes equations which, expressed in ψ and the dimensionless vorticity ζ, read as:

the dimensionless stream function ψ

$$E^2\psi = \zeta r \sin\theta \tag{9}$$

the dimensionless vorticity ζ

$$\frac{Re}{2}\left[\frac{\partial\psi}{\partial r}\frac{\partial}{\partial\theta}\left(\frac{\zeta}{r\sin\theta}\right)-\frac{\partial\psi}{\partial\theta}\frac{\partial}{\partial r}\left(\frac{\zeta}{r\sin\theta}\right)\right] = E^2(\zeta r\sin\theta) \tag{10}$$

where

$$E^2 = \frac{\partial^2}{\partial r^2}+\frac{\sin\theta}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\right). \tag{11}$$

The boundary conditions considered are:

$$\theta = 0.0 \quad \text{and} \quad \theta = \pi, \quad \psi = \zeta = 0.0 \tag{12}$$

$$r = 1, \quad \psi = \frac{\partial\psi}{\partial r} = 0.0, \quad \zeta = \frac{E^2\psi}{\sin\theta} \tag{13}$$

$$r = \kappa, \quad \psi = \tfrac{1}{2}\sin^2\theta\kappa^2 \tag{14}$$

and either zero vorticity

$$r = \kappa, \quad \zeta = 0.0 \tag{15a}$$

or free surface

$$r = \kappa, \quad \frac{\partial V_\theta}{\partial r}-\frac{V_\theta}{r} = 0.0. \tag{15b}$$

For the case of the single particle the boundary conditions (14) and (15a) with $\kappa = \infty$ are valid.

The quantities used to quantify the heat transfer results are:

the average particle temperature, $\bar{Z}_1$

$$\bar{Z}_1 = \frac{3}{2}\int_0^1 r^2\left(\int_0^\pi Z_1\sin\theta\,d\theta\right)dr \tag{16}$$

the local instantaneous Nusselt number, Nu_θ

$$Nu_\theta = -\frac{2}{\bar{Z}_1}\left(\frac{\partial Z_2}{\partial r}\right)\bigg|_{r=1} \tag{17}$$

the overall instantaneous Nusselt number, $Nu_{1(2)}$

$$Nu_2 = -\frac{2}{3}\Phi_h\frac{d\ln(\bar{Z}_1)}{d\tau_2} \quad \text{if } \Phi_\lambda \geqslant 1 \tag{18a}$$

$$Nu_1 = -\frac{2}{3}\frac{d\ln(\bar{Z}_1)}{d\tau_1} \quad \text{if } \Phi_\lambda < 1. \tag{18b}$$

The values of the Nu numbers computed with the relations (18) are compared with those provided by the additivity formula

$$\frac{1}{Nu_i} = \frac{\lambda_i}{\lambda_1}\left(\frac{1}{Nu_{in}}+\Phi_\lambda\frac{1}{Nu_{ex}}\right) \quad i = 1,2. \tag{19}$$

For a rigid sphere the value of the Nu_{in} is well known, $Nu_{in} = 6.580$. The values of Nu_{ex} are computed with the relation (5)–(25) of ref. [1].

METHOD OF SOLUTION

The energy and the Navier–Stokes equations were solved numerically. In the fluid phase the radial coordinate r was replaced by z using the transformation $r = \exp(z)$. The finite difference method was used for discretization.

The Navier–Stokes equations being uncoupled from the energy equations can be solved independently of them. The algorithm employed is the nested defect-correction multigrid iteration [17]. The method is well described in ref. [17] and it is not necessary to be reproduced here. The energy equations were solved with the algorithm employed in ref. [7]. The solid phase heat transfer equation (2) was discretized with the central second order accurate finite difference scheme. In the fluid phase, the exponentially fitted scheme [18], was used. The spatial mesh has 129×129 points in each phase for the single particle. For assemblages the number of radial steps in the continuous phase is smaller and depends on the outer sphere position. The angular step in the particle is equal to the angular step size in the continuous phase and has the value $\pi/128$. The radial step size in particle is equal to $1/128$. In the continuous phase, the radial step size has the value $\pi/128$. A decomposition procedure of the ADI type, which necessitates at every time step the solution of a tridiagonal system, is used to solve the energy equations. All the computations were done on a HP 9000 715/80 work station in FORTRAN Double Precision.

RESULTS

The results obtained for the two subjects are presented separately. The single sphere section has the aim to extend the creeping flow analysis [7] on the domain $Re > 1$. In the case of the article assemblages the analysis is focused on the specific aspects of interphase heat transfer in packed beds at low Pe numbers.

The single sphere

The main parameters of a conjugate problem are those which refer to the physical properties ratio. Usually, the rigid sphere is assimilated with a solid particle. If the environmental medium is a gas, air for example, the values of the ratio Φ_λ vary between $\mathcal{O}(1)$ for plastics to $\mathcal{O}(10^3)$ for metals and $\Phi_h \gg 1$. If the surrounding medium is a liquid, the Φ_λ values must be divided by a factor approximately equal to $\mathcal{O}(10)$ and $\Phi_h \in [0.10, 10.0]$. For a liquid–liquid system these ratios can be considered comprised between 1×10^{-1} and 1×10. In the gas–liquid systems the Φ_λ and Φ_h values are either $\gg1$ or $\ll1$. In the context of this paper, the fluid–fluid systems can be taken into consideration only if the fluid particle behaves as a rigid sphere. The conditions under a fluid particle can be considered rigid sphere are discussed in ref. [1]. So, the ratios Φ_λ and Φ_h can take values comprised between 0.0 and ∞. The

Table 1. Asymptotic *Nu* number comparison ($Re = 20.0$, $Pe = 300.0$, $\Phi_\alpha = 1.0$)

Φ_λ	Ref. [8]	Present
0.333333	1.99	1.91
1.0	4.19	4.05
3.0	6.83	6.63

variation domain considered in this paper is 1×10^{-2}–1×10^{2}.

To verify the accuracy of the present computations, the parameters sets for which Nguyen *et al.* [8] present numerical values were simulated. The comparison between the present results and those of ref. [8] are presented in Table 1. As in ref. [8], only the relation (18a) is used to compute the *Nu* number. Table 1 shows that the agreement between the two asymptotic *Nu* values is very good (the relative error is less than 5%). Also, it must be mentioned that the numerical method used in ref. [8] is different from that used here.

The results presented were obtained at two particle Reynolds numbers ($Re = 10$; 50) and *Pe* constant and equal to 100. The reasons of this choice are: at $Re = 10.0$ the flow is sufficiently far from the creeping regime and the flow separation is not present; in this way, the phenomena observed in the creeping flow regime [7] are explored in a different hydrodynamic regime in the absence of any other specific phenomenon; at $Re = 50.0$ the flow separation is fully developed and the influence of the flow separation on the conjugate heat transfer can be studied.

The values of the asymptotic *Nu* for $Re = 10.0$ and $Pe = 100.0$ are depicted in Table 2. The comparison is made with the values computed with the additivity formula (19). Table 2 shows that for $\Phi_h > 1$ the differences between the present *Nu* and the values computed with relation (19) are those usually encountered. For a given Φ_h value, the dependence *Nu* vs Φ_λ is that expected. If Φ_λ tends to zero or ∞, *Nu* tends to Nu_{in} or Nu_{ex}. An asymptotic behaviour is observed for $\Phi_h \to \infty$. A different behaviour is observed at $\Phi_h \leqslant 1$. The differences between the present *Nu* and that obtained with relation (19) increase considerably with the decrease in Φ_h. In the domain $\Phi_\lambda > 1$, the dependence *Nu* vs Φ_λ ceases to be monotone increasing. Note that for $\Phi_h < 1$ the instantaneous *Nu* attains his steady value at very low values of the particle average temperature ($Z_1 < 0.010$). To complete the information about the influence of the heat capacity ratio on the conjugate heat transfer in Figs. 1–3 the Z_1 time variation is presented. The influence of Φ_h on the conjugate heat transfer is less studied in comparison with the Φ_λ influence. For this reason, it was preferred in Figs. 1–3, to present more data about this aspect. In agreement with the *Nu* values presented in Table 2, Figs. 1–3 show that the influence of Φ_h on Z_1 time variation is less significant at $\Phi_\lambda = 0.10$ and increases considerably at $\Phi_\lambda = 1.0$ and 100.0.

The Φ_h and Φ_λ influence on the asymptotic *Nu* can be related to the thermal wake phenomenon [6]. The thermal wake is the geometrical region where the instantaneous local *Nu* number becomes negative. The thermal wake is analysed using the terminology defined in ref. [7], i.e. *thermal inversion point*, $\Phi_{\lambda,crt}$ and $\Phi_{h,crt}$. The point on the sphere surface where the first negative local *Nu* value occurs is named *thermal inversion point* (TIP). For a given Φ_h value, the critical Φ_λ value, $\Phi_{\lambda,crt}$, is defined as:

- for $\Phi_\lambda < \Phi_{\lambda,crt}$, there are no thermal inversion points;
- for $\Phi_\lambda \geqslant \Phi_{\lambda,crt}$, there is a thermal inversion point.

In a similar manner, the critical Φ_h value, $\Phi_{h,crt}$, is defined as:

- for $\Phi_h \leqslant \Phi_{h,crt}$, there is a $\Phi_{\lambda,crt}$;
- for $\Phi_h > \Phi_{h,crt}$, there is no $\Phi_{\lambda,crt}$.

The TIP steady position on the surface sphere, measured from the rear stagnation point, function of Φ_λ and Φ_h, is depicted in Fig. 4. The symbols indicate the Φ_h value. Because for $\Phi_h \geqslant 5.0$ there is no thermal wake, in Fig. 4, the cases $\Phi_h = 10.0$ and 100.0 are not plotted. Figure 4 shows the existence of the Φ_λ and Φ_h critical values, $\Phi_{h,crt} \in (2, 5)$ as in creeping flow, and puts in evidence the fact that for $\Phi_\lambda > \Phi_{\lambda,crt}$ the dimension of the thermal wake increases continuously with

Table 2. Asymptotic values of the *Nu* number at $Pe = 100.0$ and $Re = 10.0$

	Present results Φ_h									
Φ_λ	0.010	0.10	0.20	0.50	1.0	2.0	5.0	10.0	100.0	Equation (19)
0.010	6.49	6.539	6.54	6.54	6.54	6.54	6.54	6.54	6.54	6.515
0.10	0.89	5.855	6.09	6.165	6.183	6.191	6.196	6.197	6.199	5.979
0.20	0.489	4.03	5.33	5.696	5.776	5.808	5.825	5.83	5.835	5.48
0.50	0.24	1.73	2.98	4.255	4.615	4.755	4.83	4.853	4.874	4.38
1.0	0.094	0.878	1.575	2.683	3.21	3.468	3.62	3.672	3.706	3.28
2.0	0.096	0.885	1.606	2.936	3.79	4.315	4.64	4.74	4.79	4.376
5.0	0.096	0.881	1.605	3.031	4.11	4.875	5.42	5.61	5.76	5.467
10.0	0.095	0.874	1.593	3.036	4.19	5.039	5.676	5.902	6.10	5.962
100.0	0.095	0.859	1.568	3.015	4.22	5.155	5.88	6.148	6.399	6.491

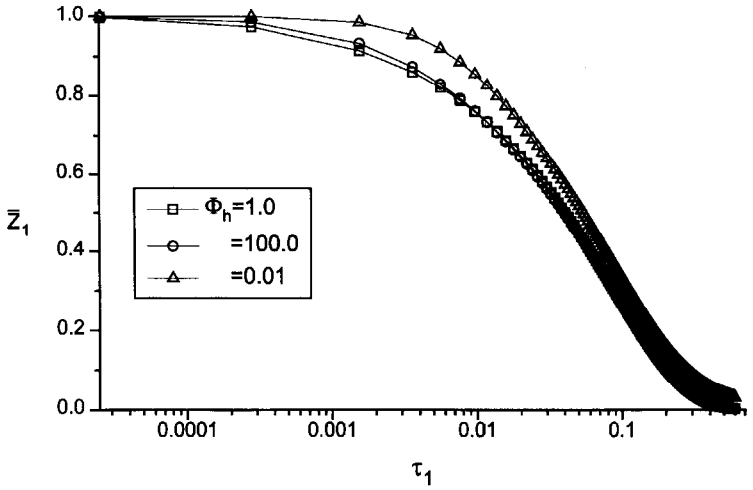

Fig. 1. Variation of the average particle temperature with the dimensionless particle time τ_1 at $Pr = 10.0$, $\Phi_\lambda = 0.10$ and $Re = 10.0$.

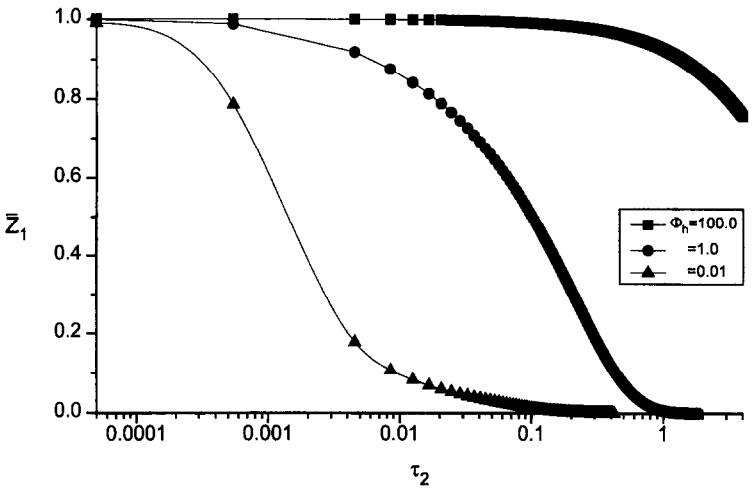

Fig. 2. Variation of the average particle temperature with the dimensionless particle time τ_2 at $Pr = 10.0$, $\Phi_\lambda = 1.0$ and $Re = 10.0$.

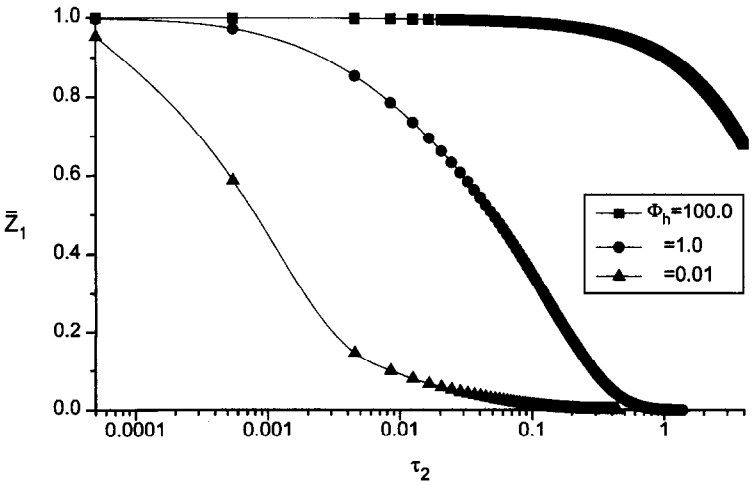

Fig. 3. Variation of the average particle temperature with the dimensionless particle time τ_2 at $Pr = 10.0$, $\Phi_\lambda = 100.0$ and $Re = 10.0$.

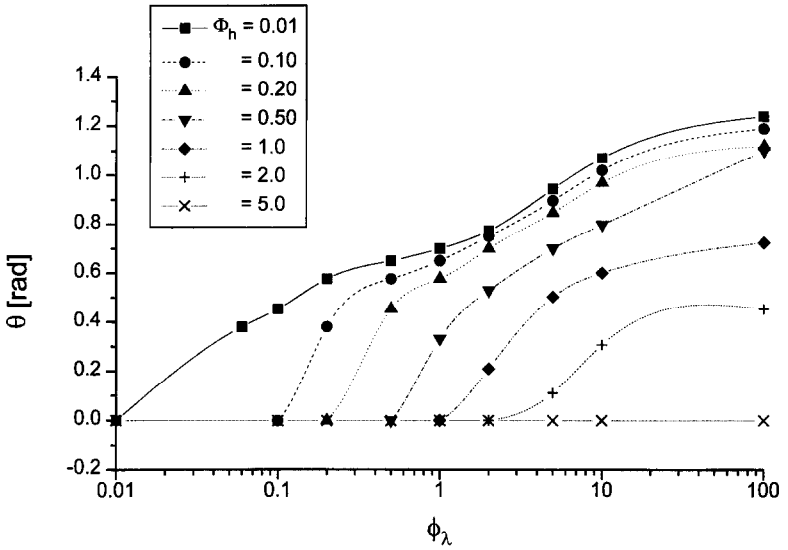

Fig. 4. The position of the thermal inversion point on the particle surface function of Φ_λ and Φ_h at $Pe = 100.0$ and $Re = 10.0$.

the increase in Φ_λ. The thermal wake dimension increases with the decrease of Φ_h and attains its maximum at $\Phi_h = 0.010$. This behaviour is opposed to that observed for asymptotic Nu. At $\Phi_h \geqslant 1$, where the influence of the heat capacity ratio on the asymptotic Nu is less significant, the thermal wake dimension reduces considerably until disappearance. The TIP values are close to those of the creeping flow. Also, the whole system behaviour resembles that encountered in creeping flow [7].

Table 3 shows the values of the asymptotic Nu at $Re = 50.0$ and $Pr = 2.0$. As in the previous case the present results are compared with those provided by the relation (19). Table 3 does not put in evidence any new and spectacular results. For $\Phi_h > 1$ the statements made previously remain valid. At $\Phi_h < 1$ and $\Phi_\lambda \geqslant 1$ the instantaneous Nu does not tend to a "frozen" steady value. The values depicted in Table 3 are attained at $\bar{Z}_1 \approx 0.0010$ when the Nu time variation becomes very small. The steady TIP position on the sphere surface at $Re = 50.0$ and $Pe = 100.0$ is shown in Fig. 5. It can be seen that the situation is similar but not identical with that depicted in Fig. 4. A TIP occurs at $\Phi_h = 5.0$ and $\Phi_\lambda = 100.0$. However, the conclusion which can be drawn is that the flow separation does not affect significantly the thermal wake phenomenon. Other cases with $Re = 50, 100, 200$ and $Pr = 5.0$ were simulated. The aspects but in evidence at $Pe = 100.0$ remain valid.

In all the computations performed until now the values assigned for the Pr number are typical for a liquid continuous phase. For gases $Pr < 1$ and for liquid metals $Pr \ll 1$. In these conditions the question which naturally arises is if the aspects put in evidence previously are valid only for high Pr number values or are generally valid, independent of the Pr number value. To clarify this problem some of the cases previously simulated with $Pr = 2.0$ were replayed with $Pr = 0.70$. The results are quantitatively different, of course, but the qualitative behaviour is the same.

Table 3. Asymptotic values of the Nu number at $Pe = 100.0$ and $Re = 50.0$

	Present results Φ_h									
Φ_λ	0.010	0.10	0.20	0.50	1.0	2.0	5.0	10.0	100.0	Equation (19)
0.010	6.232	6.542	6.543	6.543	6.543	6.543	6.543	6.543	6.543	6.52
0.10	0.67	5.28	6.083	6.188	6.21	6.22	6.223	6.225	6.226	6.035
0.20	0.409	2.65	4.80	5.722	5.82	5.858	5.876	5.882	5.887	5.573
0.50	0.21	1.22	2.12	4.024	4.66	4.843	4.93	4.955	4.977	4.533
1.0	0.058	0.567	1.096	2.344	3.204	3.575	3.755	3.809	3.836	3.458
2.0	0.061	0.595	1.155	2.548	3.786	4.517	4.921	5.046	5.15	4.689
5.0	0.066	0.634	1.234	2.754	4.232	5.258	5.902	6.12	6.308	5.965
10.0	0.079	0.667	1.279	2.858	4.409	5.532	6.275	6.53	6.759	6.559
100.0	0.095	0.76	1.33	2.967	4.572	5.773	6.612	6.905	7.176	7.206

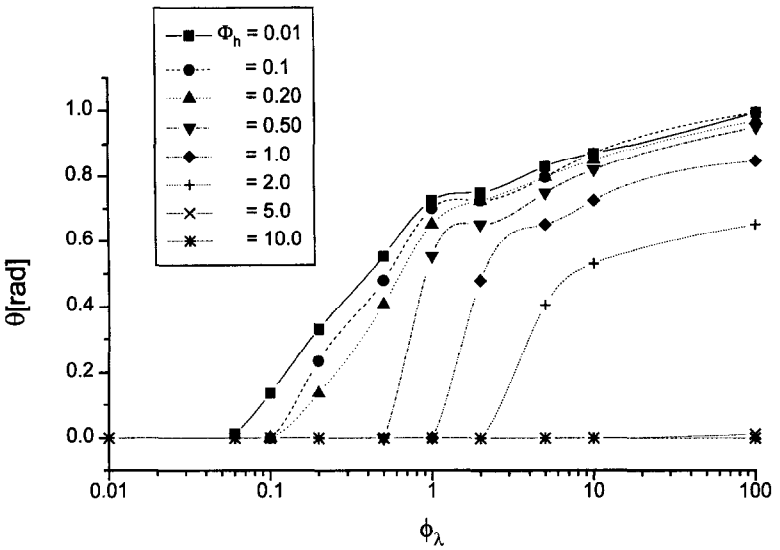

Fig. 5. The position of the thermal inversion point on the particle surface function of Φ_λ and Φ_h at $Pe = 100.0$ and $Re = 50.0$.

The particle assemblages

An extensive discussion concerning the fluid–particle systems is outside the aims of this paper. The main obstacle to be overcome in the description of such systems is that of satisfactorily treating particle–particle interactions. In the cell models the many-body problem is replaced by a simple and conceptually more attractive continuous involving only one particle. In spite of the fact that the velocity components provided by the cell models differ from those of the single particle, it is expected that, at least qualitatively, the phenomena put in evidence in the previous section to have a similar form for the multi-particle systems (of course at the same values of the *Pe* number). In this situation it is more interesting to give this section a different problem. The problem analysed is the interphase heat transfer in packed beds.

One area in which cell models appear promising is the description of steady flow through fluid–particle assemblages with voidages in the packed bed range. El-Kaissy and Homsy [9] show that in their classical formulation these models describe well the packed bed hydrodynamics in the domain $Re/(1-\varepsilon) \leqslant 40.0$.

One of the problems which is still open concerning the heat transfer in packed beds, is the interphase heat transfer coefficient at low *Pe* numbers ($Pe \leqslant 20.0$). Martin [14] explained the discrepancies between theory and experiment based on the nonuniformity of the packing. However, all the theoretical descriptions neglect the intrinsic conjugate character of the heat transfer. In all cases the particle temperature is assumed to be constant and the corresponding formula for the *Nu* number computation are of the type $Nu = 2+, \ldots$.

The starting point of the present analysis is Fig. 7 of ref. [13]. From the data presented in ref. [13] the systems analysed in refs. [19] and [20] were selected. The only reason for this selection is the availability of the data.

The results, expressed only in terms of the asymptotic Nu number, are presented in Table 4. As recommended in ref. [21], the boundary condition (6) is replaced by

$$r = \kappa, \quad \frac{\partial Z_2}{\partial r} = 0.0. \tag{20}$$

Two values are presented in the column of the present results. The left value was computed using the Happel model [11], while the right corresponds to the Kuwabara model [12]. Because the experimental values can be read only from the figures, as supplementary com-

Table 4. Asymptotic values of the *Nu* numbers at $Pr = 0.70$

Re	Φ_λ	Φ_h	Present	Experiment	Equation (19) of ref. [21]
0.01	25.0	1675.0	0.90 1.45	4×10^{-4}	0.0002
0.01	11.0	960.0	0.90 1.45	4×10^{-4}	0.0002
0.10	25.0	1675.0	0.91 1.48	5×10^{-3}	0.002
0.10	11.0	960.0	0.90 1.46	5×10^{-3}	0.002
1.0	25.0	1675.0	0.92 1.79	—	0.020
1.0	11.0	960.0	0.91 1.68	—	0.020
10.0	2.30	1380.0	1.35 3.01	0.40	0.201

parison criteria the results provided by the relation (19) of ref. [21] are used. Table 4 leads to the following statements:

- the Kuwabara model gives higher values than the Happel model;
- in an unexpected way, in the limit $Pe \to 0$ Nu tends to a finite value higher than 0 (it is well known that in the case of conjugate heat transfer the asymptotic Nu tends to zero when Pe tends to zero);
- the experimental data are not well approximated by the theoretical computations.

The previous statements cannot be considered the final verdict concerning the ability of the cell models to describe the interphase heat transfer in packed beds. The boundary condition for temperature at $r = \kappa$ can be viewed as an open problem.

CONCLUSIONS

Two problems arising from the conjugate unsteady heat transfer in fluid–particle systems were studied. The first refers to the single particle in steady laminar flow at moderate Re number values. The main aspect analysed is the effect of the parameters Φ_λ and Φ_h on heat transfer. The results obtained show that Φ_h and Φ_λ have a strong influence on the conjugate heat transfer. The system behaviour is close to that observed in creeping flow [7]. At moderate Pe numbers, the occurrence and the development of the thermal wake are not significantly affected by the hydrodynamic regime.

The second problem analysed has as subject the particle assemblages. The cell models were employed to describe the flow. Due to the following facts:

- the cell models describe well the packed bed hydrodynamics at low Re numbers;
- the usual theoretical relations used to compute the interphase heat transfer in packed beds fail for $Pe < 20.0$,

the second problem studied is focused on the conjugate heat transfer at low Pe numbers in packed beds. Two sets of experimental data were chosen for comparison. The theoretical predictions of the cell models do not reproduce the experiments.

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